

[不定積分 II]4H1 前半

1. 次の不定積分を求めよ。

$$(1) \int \frac{1}{x^2+25} dx$$

$$(2) \int \frac{1}{x^2-25} dx$$

$$(3) \int \frac{1}{\sqrt{x^2+25}} dx$$

$$(4) \int \frac{1}{\sqrt{25-x^2}} dx$$

$$(5) \int \frac{1}{\sqrt{x^2-25}} dx$$

$$(6) \int \sqrt{x^2+25} dx$$

$$(7) \int \sqrt{25-x^2} dx$$

$$(8) \int \frac{1}{x^2+2x+5} dx$$

$$(9) \int \frac{1}{x^2+2x-3} dx$$

$$(10) \int \frac{1}{\sqrt{x^2+2x+5}} dx$$

$$(11) \int \frac{1}{\sqrt{3-x^2-2x}} dx$$

[部分分数分解]

2. 次の不定積分を求めよ。

$$(1) I = \int \frac{5}{(x-1)(x+3)} dx$$

$$(2) I = \int \frac{9x-4}{(3x+1)(x-2)} dx$$

$$(3) I = \int \frac{x^2+3x}{(x^2+1)(x+1)} dx$$

$$(4) I = \int \frac{x-3}{(x-1)^2(x+1)} dx$$

[不定積分 II] 4H1 前半

1. 次の不定積分を求めよ。

$$(1) \int \frac{1}{x^2+25} dx = \frac{1}{5} \tan^{-1} \frac{x}{5} + C$$

$$(2) \int \frac{1}{x^2-25} dx = \frac{1}{10} \log \left| \frac{x-5}{x+5} \right| + C$$

$$(3) \int \frac{1}{\sqrt{x^2+25}} dx = \log \left| x + \sqrt{x^2+25} \right| + C$$

$$(4) \int \frac{1}{\sqrt{25-x^2}} dx = \sin^{-1} \frac{x}{5} + C$$

$$(5) \int \frac{1}{\sqrt{x^2-25}} dx = \log \left| x + \sqrt{x^2-25} \right| + C$$

$$(6) \int \sqrt{x^2+25} dx = \frac{1}{2} \left( x\sqrt{x^2+25} + 25 \log \left| x + \sqrt{x^2+25} \right| \right) + C$$

$$(7) \int \sqrt{25-x^2} dx = \frac{1}{2} \left( x\sqrt{25-x^2} + 25 \sin^{-1} \frac{x}{5} \right) + C$$

$$(8) \int \frac{1}{x^2+2x+5} dx = \int \frac{1}{\underbrace{(x^2+2x+1)}_{(x+1)^2} - 1 + 5} dx$$

$$= \int \frac{1}{(x+1)^2+4} dx = \frac{1}{2} \tan^{-1} \frac{x+1}{2} + C$$

$$(9) \int \frac{1}{x^2+2x-3} dx = \int \frac{1}{\underbrace{(x^2+2x+1)}_{(x+1)^2} - 1 - 3} dx$$

$$= \int \frac{1}{(x+1)^2-4} dx = \frac{1}{2 \cdot 2} \log \left| \frac{(x+1)-2}{(x+1)+2} \right| + C$$

$$= \frac{1}{4} \log \left| \frac{x-1}{x+3} \right| + C$$

$$(10) \int \frac{1}{\sqrt{x^2+2x+5}} dx = \int \frac{1}{\sqrt{(x^2+2x+1)-1+5}} dx$$

$$= \int \frac{1}{\sqrt{(x+1)^2+4}} dx = \log \left| (x+1) + \sqrt{(x+1)^2+4} \right| + C$$

$$= \int \frac{1}{\sqrt{(x+1)^2+4}} dx = \log \left| x+1 + \sqrt{x^2+2x+5} \right| + C$$

$$(11) \int \frac{1}{\sqrt{3-x^2-2x}} dx = \int \frac{1}{\sqrt{3+1-(x^2+2x+1)}} dx$$

$$= \int \frac{1}{\sqrt{4-(x+1)^2}} dx = \sin^{-1} \frac{x+1}{2} + C$$

[部分分数分解]

2. 次の不定積分を求めよ。

$$(1) I = \int \frac{5}{(x-1)(x+3)} dx$$

$$\frac{5}{(x-1)(x+3)} = \frac{a}{x-1} + \frac{b}{x+3} = \frac{(a+b)x+3a-b}{(x-1)(x+3)} \text{ より}$$

$$a+b=0, 3a-b=5 \Rightarrow a=\frac{5}{4}, b=-\frac{5}{4} \quad \text{だから、}$$

$$I = \frac{5}{4} \int \left( \frac{1}{x-1} - \frac{1}{x+3} \right) dx = \frac{5}{4} \log \left| \frac{x-1}{x+3} \right| + C$$

$$(2) I = \int \frac{9x-4}{(3x+1)(x-2)} dx$$

$$\frac{9x-4}{(3x+1)(x-2)} = \frac{a}{3x+1} + \frac{b}{x-2} = \frac{(a+3b)x-2a+b}{(3x+1)(x-2)} \text{ より}$$

$$a+3b=9, -2a+b=-4 \Rightarrow a=3, b=2 \quad \text{だから、}$$

$$I = \int \left( \frac{3}{3x+1} + 2 \cdot \frac{1}{x-2} \right) dx = \log \left| \underbrace{3x+1}_{\log|3x+1|+2\log|x-2|} (x-2)^2 \right| + C$$

$$(3) I = \int \frac{x^2+3x}{(x^2+1)(x+1)} dx$$

$$\frac{x^2+3x}{(x^2+1)(x+1)} = \frac{ax+b}{x^2+1} + \frac{c}{x+1} = \frac{(a+c)x^2+(a+b)x+(b+c)}{(x^2+1)(x+1)}$$

$$a+c=1, a+b=3, b+c=0 \Rightarrow a=2, b=1, c=-1 \text{ より}$$

$$I = \int \left( \frac{2x+1}{x^2+1} - \frac{1}{x+1} \right) dx = \int \left( \frac{1}{x^2+1} + \frac{2x}{x^2+1} - \frac{1}{x+1} \right) dx$$

$$= \tan^{-1} x + \log \left| \frac{x^2+1}{x+1} \right| + C$$

$$(4) I = \int \frac{x-3}{(x-1)^2(x+1)} dx$$

$$\frac{x-3}{(x-1)^2(x+1)} = \frac{a}{(x-1)^2} + \frac{b}{x-1} + \frac{c}{x+1}$$

$$= \frac{(b+c)x^2+(a-2c)x+(a-b+c)}{(x-1)^2(x+1)}$$

$$b+c=0, a-2c=1, a-b+c=-3 \Rightarrow a=-1, b=1, c=-1$$

$$I = \int \left( \frac{-1}{(x-1)^2} + \frac{1}{x-1} - \frac{1}{x+1} \right) dx$$

$$= \int \left( -(x-1)^{-2} + \frac{1}{x-1} - \frac{1}{x+1} \right) dx = \frac{1}{x-1} + \log \left| \frac{x-1}{x+1} \right| + C$$