演習　定積分_第 07 回

学年[2]年　学科[MI・AC・BC]　番号[　　　　]　氏名　[　　　　　　　　　　]

問 10.17　区分求積法を用いて $\int_0^2 x\,dx$ の値を求めよ。

手順１：分割の幅　$\Delta x = \dfrac{b-a}{n} = \dfrac{2-0}{n} = \dfrac{2}{n}$

一 般 項　$x_k = a + k\Delta x = 0 + k \cdot \dfrac{2}{n} = \dfrac{2k}{n}$

関　　数　$y = f(x) = x \quad \left(\Rightarrow \quad f(x_k) = x_k = \dfrac{2k}{n} \right)$

手順２：小長方形の面積　$S_k = f(x_k)\Delta x = \dfrac{2k}{n} \cdot \dfrac{2}{n} = \dfrac{4k}{n^2}$

手順３：近似値　$S \fallingdotseq \sum_{k=1}^{n} S_k = \sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n} \dfrac{4k}{n^2} = \dfrac{4}{n^2}\sum_{k=1}^{n} k = \dfrac{4}{n^2} \times \dfrac{1}{2} n(n+1) = \dfrac{2(n+1)}{n}$

手順４：面　積　$S = \lim_{n\to+\infty} \sum_{k=1}^{n} S_k = \lim_{n\to+\infty} \dfrac{2(n+1)}{n} = \lim_{n\to+\infty} \dfrac{2n+2}{n} = \lim_{n\to+\infty} \left(2 + \dfrac{2}{n}\right) = 2$

問 10.18　次の極限の値を求めよ。

(1) $\lim_{n\to+\infty} \left(\dfrac{1}{n^2+1^2} + \dfrac{2}{n^2+2^2} + \dfrac{3}{n^2+3^2} + \cdots + \dfrac{n}{n^2+n^2} \right)$

$$= \lim_{n\to+\infty} \sum_{k=1}^{n} \frac{k}{n^2+k^2} = \lim_{n\to+\infty} \frac{1}{n}\sum_{k=1}^{n} \frac{nk}{n^2+k^2} = \lim_{n\to+\infty} \frac{1}{n}\sum_{k=1}^{n} \frac{\left(k/n\right)}{1+\left(k/n\right)^2}$$

$$= \int_0^1 \frac{x}{1+x^2}\,dx = \frac{1}{2}\int_0^1 \frac{2x}{1+x^2}\,dx = \frac{1}{2}\Big[\log(1+x^2)\Big]_0^1 = \frac{1}{2}(\log 2 - \log 1) = \frac{1}{2}\log 2$$

(2) $\lim_{n\to+\infty} \dfrac{1}{n^3}\left\{(n+1)^2 + (n+2)^2 + (n+3)^2 + \cdots (n+n)^2\right\}$

$$= \lim_{n\to+\infty} \left\{ \frac{(n+1)^2}{n^3} + \frac{(n+2)^2}{n^3} + \frac{(n+3)^2}{n^3} + \cdots + \frac{(n+n)^2}{n^3} \right\}$$

$$= \lim_{n\to+\infty} \sum_{k=1}^{n} \frac{(n+k)^2}{n^3} = \lim_{n\to+\infty} \frac{1}{n}\sum_{k=1}^{n} \frac{n(n+k)^2}{n^3} = \lim_{n\to+\infty} \frac{1}{n}\sum_{k=1}^{n} \left(\frac{n+k}{n}\right)^2 = \lim_{n\to+\infty} \frac{1}{n}\sum_{k=1}^{n} \left(1+\frac{k}{n}\right)^2$$

$$= \int_0^1 (1+x)^2\,dx = \left[\frac{1}{1} \times \frac{1}{3}(1+x)^3\right]_0^1 = \frac{1}{3}(2^3 - 1^3) = \frac{7}{3}$$