演習　定積分_第 06 回

学年［ 2 ］年　学科［ MI・AC・BC ］　番号［　　　］　氏名　［　　　　　　　　　　］

問 10.5　次の定積分を計算せよ。（前頁の【注意事項】は読みましたか？）

(1) $\int_0^{\pi} \cos^3 x\ dx = 0$

(2) $\int_0^{\pi} \cos^4 x\ dx = 2\int_0^{\frac{\pi}{2}} \cos^4 x\ dx = 2\times\left(\frac{3}{4}\times\frac{1}{2}\times\frac{\pi}{2}\right) = \frac{3}{8}\pi$

(3) $\int_0^{\pi} \sin^5 x\ dx = 2\int_0^{\frac{\pi}{2}} \sin^5 x\ dx = 2\times\left(\frac{4}{5}\times\frac{2}{3}\times 1\right) = \frac{16}{15}$

問 10.6　次の定積分を求めよ。

(1) $\int_0^2 x^2\sqrt{x^3+1}\ dx$

$u = x^3+1 \cdots ①$　とおくと　$\frac{du}{dx} = 3x^2$　　$\therefore dx = \frac{du}{3x^2} \cdots ②$

$x: 0 \to 2$　のとき　$u: 1 \to 9 \cdots ③$

①～③より　（与式）$= \int_1^9 \cancel{x^2}\sqrt{u}\times\frac{du}{3\cancel{x^2}} = \frac{1}{3}\int_1^9 u^{\frac{1}{2}}\ du$

$= \frac{1}{3}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^9 = \frac{2}{9}\left[\sqrt{u^3}\right]_1^9 = \frac{2}{9}\left[u\sqrt{u}\right]_1^9 = \frac{2}{9}\left(9\sqrt{9}-1\right) = \frac{52}{9}$

(2) $\int_1^e \frac{(\log x)^4}{x}dx$

$u = \log x \cdots ①$　とおくと　$\frac{du}{dx} = \frac{1}{x}$　　$\therefore dx = x\ du \cdots ②$

$x: 1 \to e$　のとき　$u: \log 1 = 0 \to \log e = 1 \cdots ③$

①～③より　（与式）$= \int_0^1 \frac{u^4}{\cancel{x}}\times\cancel{x}\ du = \int_0^1 u^4\ du = \left[\frac{1}{5}u^5\right]_0^1 = \frac{1}{5}$

(3) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}\ dx$

$u = \cos x \cdots ①$　とおくと　$\frac{du}{dx} = -\sin x$　　$\therefore dx = \frac{du}{-\sin x} \cdots ②$

$x: 0 \to \pi/2$　のとき　$u: \cos 0 = 1 \to \cos(\pi/2) = 0 \cdots ③$

①～③より　（与式）$= \int_1^0 \frac{\cancel{\sin x}}{1+u^2}\times\frac{du}{-\cancel{\sin x}} = -\int_1^0 \frac{1}{1+u^2}\ du$

$= -\left[\mathrm{Tan}^{-1}(u)\right]_1^0 = -\left\{\mathrm{Tan}^{-1}(0) - \mathrm{Tan}^{-1}(1)\right\} = -\left(0-\frac{\pi}{4}\right) = \frac{\pi}{4}$