演習　定積分_第 04 回

学年[2]年　学科[MI・AC・BC]　番号[　　　　]　氏名　[　　　　　　　　　　　　]

問 10.1　次の定積分を計算せよ。

(1) $\displaystyle\int_{-1}^{2} x^4\, dx = \left[\frac{1}{5}x^5\right]_{-1}^{2} = \frac{1}{5}\{32-(-1)\} = \frac{33}{5}$

(2) $\displaystyle\int_{1}^{4} \sqrt{x}\, dx = \int_{1}^{4} x^{\frac{1}{2}}\, dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_{1}^{4} = \left[\frac{2}{3}\sqrt{x^3}\right]_{1}^{4} = \left[\frac{2}{3}x\sqrt{x}\right]_{1}^{4}$

$\displaystyle= \frac{2}{3}(4\sqrt{4}-1) = \frac{2}{3}(8-1) = \frac{14}{3}$

(3) $\displaystyle\int_{1}^{4} \frac{1}{\sqrt{x}}\, dx = \int_{1}^{4} x^{-\frac{1}{2}}\, dx = \left[2x^{\frac{1}{2}}\right]_{1}^{4} = \left[2\sqrt{x}\right]_{1}^{4}$

$= 2(\sqrt{4}-1) = 2(2-1) = 2$

(4) $\displaystyle\int_{1}^{e} \frac{1}{x}\, dx = \left[\log|x|\right]_{1}^{e} = \log e - \log 1 = 1 - 0 = 1$

(5) $\displaystyle\int_{0}^{1} e^{2x}\, dx = \left[\frac{1}{2}e^{2x}\right]_{0}^{1} = \frac{1}{2}(e^2 - e^0) = \frac{e^2-1}{2}$

(6) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos 2x\, dx = \left[\frac{1}{2}\sin 2x\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}} = \frac{1}{2}\left(\sin\frac{\pi}{2} - \sin\frac{\pi}{3}\right)$

$\displaystyle= \frac{1}{2}\left(1 - \frac{\sqrt{3}}{2}\right) = \frac{2-\sqrt{3}}{4}$

(7) $\displaystyle\int_{0}^{\frac{\pi}{3}} \frac{1}{\cos^2 x}\, dx = \left[\tan x\right]_{0}^{\frac{\pi}{3}} = \tan\frac{\pi}{3} - \tan 0 = \sqrt{3} - 0 = \sqrt{3}$

(8) $\displaystyle\int_{0}^{2} \frac{1}{x^2+4}\, dx = \left[\frac{1}{2}\mathrm{Tan}^{-1}\left(\frac{x}{2}\right)\right]_{0}^{2} = \frac{1}{2}\left\{\mathrm{Tan}^{-1}(1) - \mathrm{Tan}^{-1}(0)\right\} = \frac{1}{2}\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{8}$

(9) $\displaystyle\int_{0}^{2} \frac{x}{x^2+4}\, dx = \frac{1}{2}\int_{0}^{2} \frac{2x}{x^2+4}\, dx = \frac{1}{2}\left[\log(x^2+4)\right]_{0}^{2}$

$\displaystyle= \frac{1}{2}(\log 8 - \log 4) = \frac{1}{2}\log\frac{8}{4} = \frac{1}{2}\log 2$