演習　不定積分２_第 03 回

学年[２]年　学科[MI・AC・BC]　番号[　　　　]　氏名　[　　　　　　　　　　　　]

問 9. 14　不定積分 $\int \sqrt{1-x^2}\,dx$ を求めよ。

$$\int \sqrt{1-x^2}\,dx = \int (x)'\sqrt{1-x^2}\,dx$$

$$= x\sqrt{1-x^2} - \int \left(x \times \frac{-2x}{2\sqrt{1-x^2}} \right) dx$$

$$= x\sqrt{1-x^2} - \int \frac{-x^2}{\sqrt{1-x^2}}dx = x\sqrt{1-x^2} - \int \frac{(1-x^2)-1}{\sqrt{1-x^2}}dx$$

$$= x\sqrt{1-x^2} - \int \frac{(1-x^2)}{\sqrt{1-x^2}}\,dx + \int \frac{1}{\sqrt{1-x^2}}dx$$

$$= x\sqrt{1-x^2} - \int \sqrt{1-x^2}\,dx + \mathrm{Sin}^{-1}x$$

よって　$2\int \sqrt{1-x^2}\,dx = x\sqrt{1-x^2} + \mathrm{Sin}^{-1}x + c$

$$\therefore \int \sqrt{1-x^2}\,dx = \frac{1}{2}\left(x\sqrt{1-x^2} + \mathrm{Sin}^{-1}x \right) + C \quad （但し \ C = c/2）$$

問 9. 15　次の不定積分を求めよ。　(1) $\int \frac{1}{1-\cos x}dx$　　(2) $\int \frac{1}{2+\cos x}dx$

(0) $u = \tan\frac{x}{2}$　とおくと　$\cos x = \frac{1-u^2}{1+u^2}$，　$dx = \frac{2}{1+u^2}du$

(1) $\int \frac{1}{1-\cos x}dx = \int \frac{1}{1-\left(\frac{1-u^2}{1+u^2}\right)} \times \frac{2}{1+u^2}du = \int \frac{2}{(1+u^2)-(1-u^2)}\,du$

$$= \int \frac{2}{2u^2}\,du = \int u^{-2}du = -u^{-1} + C$$

$$= -\frac{1}{\tan(x/2)} + C \quad (= -\cot(x/2) + C)$$

(2) $\int \frac{1}{2+\cos x}dx = \int \frac{1}{2+\left(\frac{1-u^2}{1+u^2}\right)} \times \frac{2}{1+u^2}du = \int \frac{2}{2(1+u^2)+(1-u^2)}\,du$

$$= \int \frac{2}{u^2+3}\,du = 2\int \frac{1}{u^2+3}\,du = 2 \times \frac{1}{\sqrt{3}}\mathrm{Tan}^{-1}\left(\frac{u}{\sqrt{3}}\right) + C$$

$$= \frac{2}{\sqrt{3}}\mathrm{Tan}^{-1}\left\{\frac{\tan(x/2)}{\sqrt{3}}\right\} + C$$