演習　定積分_第 11 回

学年［ 2 ］年　学科［ MI・AC・BC ］　番号［　　　］　氏名　［　　　　　　　　　　］

問 10. 23　曲線 $y=\dfrac{x^2}{8}-\log x$　（$1 \leqq x \leqq e$）の長さ L を求めよ。

① $y' = f('x) = \dfrac{x}{4}-\dfrac{1}{x}$

② $1+\{f('x)\}^2 = 1+\left(\dfrac{x}{4}-\dfrac{1}{x}\right)^2 = 1+\left(\dfrac{x^2}{16}-\dfrac{1}{2}+\dfrac{1}{x^2}\right)=\dfrac{x^2}{16}+\dfrac{1}{2}+\dfrac{1}{x^2}=\left(\dfrac{x}{4}+\dfrac{1}{x}\right)^2$

③ $L=\displaystyle\int_a^b \sqrt{1+\{f'(x)\}^2}\,dx=\int_1^e\left(\dfrac{x}{4}+\dfrac{1}{x}\right)dx=\left[\dfrac{1}{8}x^2+\log x\right]_1^e$

$$=\left(\frac{1}{8}e^2+\log e\right)-\left(\frac{1}{8}+\log 1\right)=\frac{1}{8}e^2+1-\frac{1}{8}=\frac{1}{8}e^2+\frac{7}{8}$$

問 10. 24　次の広義積分を計算せよ。

(1) $\displaystyle\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx=\lim_{\varepsilon\to 0}\int_0^{1-\varepsilon}\frac{1}{\sqrt{1-x^2}}\,dx=\lim_{\varepsilon\to 0}\left[\mathrm{Sin}^{-1}(x)\right]_0^{1-\varepsilon}$

$$=\lim_{\varepsilon\to 0}\mathrm{Sin}^{-1}(1-\varepsilon)=\mathrm{Sin}^{-1}(1)=\frac{\pi}{2}$$

(2) $\displaystyle\int_1^2 \frac{1}{\sqrt{x^2-1}}\,dx=\lim_{\varepsilon\to 0}\int_{1+\varepsilon}^{2}\frac{1}{\sqrt{x^2-1}}\,dx=\lim_{\varepsilon\to 0}\left[\log|x+\sqrt{x^2-1}|\right]_{1+\varepsilon}^{2}$

$$=\log(2+\sqrt{3})-\lim_{\varepsilon\to 0}\left[\log\left\{(1+\varepsilon)+\sqrt{(1+\varepsilon)^2-1}\right\}\right]$$

$$=\log(2+\sqrt{3})-\log 1=\log(2+\sqrt{3})$$

(3) $\displaystyle\int_0^\infty \frac{1}{x^2+1}\,dx=\lim_{M\to\infty}\int_0^M\frac{1}{x^2+1}\,dx=\lim_{M\to\infty}\left[\mathrm{Tan}^{-1}x\right]_0^M$

$$=\lim_{M\to\infty}\mathrm{Tan}^{-1}M=\frac{\pi}{2}$$

(4) $\displaystyle\int_0^\infty e^{-x}\,dx=\lim_{M\to\infty}\int_0^M e^{-x}\,dx=\lim_{M\to\infty}\left[-e^{-x}\right]_0^M$

$$=\lim_{M\to\infty}\left(-e^{-M}+1\right)=\lim_{M\to\infty}\left(1-\frac{1}{e^M}\right)=1-0=1$$