演習　不定積分２_第 01 回

学年[２]年　学科[MI・AC・BC]　番号[　　]　氏名　[　　]

問 9.10　次の不定積分を求めよ。

(1) $\displaystyle\int \frac{1}{x^2-4}\,dx = \frac{1}{4}\log\left|\frac{x-2}{x+2}\right| + C$

(2) $\displaystyle\int \frac{1}{x^2+4}\,dx = \frac{1}{2}\mathrm{Tan}^{-1}\left(\frac{x}{2}\right) + C$

(3) $\displaystyle\int \frac{1}{\sqrt{4-x^2}}\,dx = \mathrm{Sin}^{-1}\left(\frac{x}{2}\right) + C$

(4) $\displaystyle\int \frac{1}{\sqrt{x^2+4}}\,dx = \log|x+\sqrt{x^2+4}| + C$

(5) $\displaystyle\int \frac{1}{\sqrt{5+4x-x^2}}\,dx$

平方完成：$5+4x-x^2 = -\{x^2-4x-5\} = -\{(x-2)^2-4-5\}$

$= -\{(x-2)^2-9\} = 9-(x-2)^2$

よって　(与式) $\displaystyle= \int \frac{1}{\sqrt{9-(x-2)^2}}\,dx = \mathrm{Sin}^{-1}\left(\frac{x-2}{3}\right) + C$

問 9.11　不定積分 $\displaystyle\int \frac{x}{\sqrt{x-1}}\,dx$ を求めよ。

$u = \sqrt{x-1}\cdots①$ とおくと　$u^2 = x-1$　　$\therefore x = u^2+1\cdots②$

②より　$\displaystyle\frac{dx}{du} = 2u$　　$\therefore dx = 2u\,du\cdots③$

①～③より　(与式) $\displaystyle= \int \frac{u^2+1}{u}\times 2u\,du = \int (2u^2+2)\,du$

$\displaystyle= \frac{2}{3}u^3 + 2u + C = \frac{2}{3}(u^2+3)u + C$

$\displaystyle= \frac{2}{3}\{(x-1)+3\}\sqrt{x-1} + C$

$\displaystyle= \frac{2}{3}(x+2)\sqrt{x-1} + C$