演習　不定積分_第 12 回

学年［ 2 ］年　学科［ MI・AC・BC ］　番号［　　　］　氏名　［　　　　　　　　　　］

問 9.9　次の不定積分を求めよ。

(01) $\int \sin 2x\, dx = -\frac{1}{2}\cos 2x + C$

(02) $\int \sin^2 x\, dx = \frac{1}{2}\int (1-\cos 2x)\, dx$

$= \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C$

$= \frac{1}{2}x - \frac{1}{4}\sin 2x + C$

(03) $\int \sin x \cos x\, dx = \frac{1}{2}\int \sin 2x\, dx$

$= -\frac{1}{4}\cos 2x + C$

(04) $\int \sin 2x \cos x\, dx$

$= \frac{1}{2}\int (\sin 3x + \sin x)\, dx$

$= \frac{1}{2}\left(-\frac{1}{3}\cos 3x - \cos x\right) + C$

$= -\frac{1}{6}\cos 3x - \frac{1}{2}\cos x + C$

(05) $\int \sin^2 x \cos x\, dx$

$u = \sin x$ …① とおくと

$\frac{du}{dx} = \cos x$　　∴ $dx = \frac{du}{\cos x}$ …②

①，②より

(与式) $= \int u^2 \cancel{\cos x} \times \frac{du}{\cancel{\cos x}} = \int u^2\, du$

$= \frac{1}{3}u^3 + C$

$= \frac{1}{3}\sin^3 x + C$

(06) $\int \frac{\cos x}{\sin^2 x}\, dx$

$u = \sin x$ …① とおくと

$\frac{du}{dx} = \cos x$　　∴ $dx = \frac{du}{\cos x}$ …②

①，②より

(与式) $= \int \frac{\cancel{\cos x}}{u^2} \times \frac{du}{\cancel{\cos x}} = \int u^{-2}\, du$

$= -u^{-1} + C$

$= \frac{-1}{\sin x} + C$

(07) $\int \frac{1}{\sin^2 x}\, dx = -\cot x + C$

(08) $\int \frac{\cos x}{\sin x}\, dx = \log(\sin x) + C$

(09) $\int \frac{\cos x}{1+\sin x}\, dx = \log(1+\sin x) + C$

(10) $\int \tan x\, dx = \int \frac{\sin x}{\cos x}\, dx$

$= -\int \frac{-\sin x}{\cos x}\, dx$

$= -\log(\cos x) + C$

(11) $\int \tan^2 x\, dx = \int \left(\frac{1}{\cos^2 x} - 1\right)\, dx$

$= \tan x - x + C$

(12) $\int x\cos 2x\, dx = \int x\left(\frac{1}{2}\sin 2x\right)'\, dx$

$= \frac{1}{2}x\sin 2x - \frac{1}{2}\int \sin 2x\, dx$

$= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C$

(13) $\int x\cos(x^2+1)\ dx$

$u = x^2+1 \cdots ①$ とおくと

$\frac{du}{dx} = 2x \qquad \therefore dx = \frac{du}{2x} \cdots ②$

①，②より

(与式) $= \int \cancel{x} \cos u \times \frac{du}{2\cancel{x}} = \frac{1}{2}\int \cos u\ du$

$= \frac{1}{2}\sin u + C$

$= \frac{1}{2}\sin(x^2+1) + C$

(14) $\int e^{-x}\cos 2x\ dx = \int (-e^{-x})'\cos 2x\ dx$

$= -e^{-x}\cos 2x - 2\int e^{-x}\sin 2x\ dx$

$= -e^{-x}\cos 2x - 2\int (-e^{-x})'\sin 2x\ dx$

$= -e^{-x}\cos 2x - 2\left(-e^{-x}\sin 2x + 2\int e^{-x}\cos 2x\ dx\right)$

$= -e^{-x}\cos 2x + 2e^{-x}\sin 2x - 4\int e^{-x}\cos 2x\ dx$

よって

$5\int e^{-x}\cos 2x\ dx = -e^{-x}\cos 2x + 2e^{-x}\sin 2x + c$

$\therefore \int e^{-x}\cos 2x\ dx = -\frac{1}{5}e^{-x}\left(\cos 2x - 2\sin 2x\right) + C$

（但し　$C = c/5$）

(15) $\int \cos 3x \cos 2x\ dx$

$= \frac{1}{2}\int (\cos 5x + \cos x)\ dx$

$= \frac{1}{2}\left(\frac{1}{5}\sin 5x + \sin x\right) + C$

$= \frac{1}{10}\sin 5x + \frac{1}{2}\sin x + C$

(16) $\int (2x+1)^3\ dx = \frac{1}{2}\times\frac{1}{4}(2x+1)^4 + C$

$= \frac{1}{8}(2x+1)^4 + C$

(17) $\int x(x^2+1)^3\ dx$

$u = x^2+1 \cdots ①$ とおくと

$\frac{du}{dx} = 2x \qquad \therefore dx = \frac{du}{2x} \cdots ②$

①，②より

(与式) $= \int \cancel{x} u^3 \times \frac{du}{2\cancel{x}} = \frac{1}{2}\int u^3\ du$

$= \frac{1}{8}u^4 + C$

$= \frac{1}{8}(x^2+1)^4 + C$

(18) $\int \frac{1}{\sqrt{2x+1}}\ dx = \int (2x+1)^{-\frac{1}{2}}\ dx$

$= \frac{1}{2}\times 2(2x+1)^{\frac{1}{2}} + C$

$= \sqrt{2x+1} + C$

(19) $\int \frac{x}{\sqrt{x^2+1}}\ dx$

$u = x^2+1 \cdots ①$ とおくと

$\frac{du}{dx} = 2x \qquad \therefore dx = \frac{du}{2x} \cdots ②$

①，②より

(与式) $= \int \frac{\cancel{x}}{\sqrt{u}} \times \frac{du}{2\cancel{x}} = \frac{1}{2}\int u^{-\frac{1}{2}}\ du$

$= u^{\frac{1}{2}} + C$

$= \sqrt{x^2+1} + C$

(20) $\int \frac{x}{x^2+1}\ dx = \frac{1}{2}\int \frac{2x}{x^2+1}\ dx$

$= \frac{1}{2}\log(x^2+1) + C$

(21) $\int (2x+1)(x^2+x+1)\ dx$

$$= \int (2x^2+2x^2+2x+x^2+x+1)\ dx$$

$$= \int (2x^3+3x^2+3x+1)\ dx$$

$$= \frac{1}{2}x^4+x^3+\frac{3}{2}x^2+x+C$$

(22) $\int (2x+1)(x^2+x+1)^4\ dx$

$u = x^2+x+1 \cdots ①$ とおくと

$\frac{du}{dx} = 2x+1 \qquad \therefore dx = \frac{du}{2x+1} \cdots ②$

①，②より

(与式) $= \int \cancel{(2x+1)}\ u^4 \times \frac{du}{\cancel{(2x+1)}} = \int u^4\ du$

$$= \frac{1}{5}u^5 + C$$

$$= \frac{1}{5}(x^2+x+1)^5 + C$$

(23) $\int \frac{2x+1}{x^2+x+1}\ dx = \log(x^2+x+1)+C$

(24) $\int \frac{2x+1}{(x^2+x+1)^4}\ dx$

$u = x^2+x+1 \cdots ①$ とおくと

$\frac{du}{dx} = 2x+1 \qquad \therefore dx = \frac{du}{2x+1} \cdots ②$

①，②より

(与式) $= \int \frac{\cancel{(2x+1)}}{u^4} \times \frac{du}{\cancel{(2x+1)}} = \int u^{-4}\ du$

$$= \frac{-1}{3}u^{-3} + C$$

$$= \frac{-1}{3(x^2+x+1)^3} + C$$

(25) $\int \frac{1}{\sqrt[3]{2x+1}}\ dx = \int (2x+1)^{-\frac{1}{3}}\ dx$

$$= \frac{1}{2} \times \frac{3}{2}(2x+1)^{\frac{2}{3}} + C = \frac{3}{4}(2x+1)^{\frac{2}{3}} + C$$

$$= \frac{3}{4}\sqrt[3]{(2x+1)^2} + C$$

(26) $\int \frac{1}{2x+1}\ dx = \frac{1}{2}\int \frac{2}{2x+1}\ dx$

$$= \frac{1}{2}\log(2x+1) + C$$

(27) $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2\ dx = \int \left(x+2+\frac{1}{x}\right)\ dx$

$$= \frac{1}{2}x^2 + 2x + \log x + C$$

(28) $\int \left(x+\frac{1}{x}\right)^2\ dx = \int \left(x^2+2+\frac{1}{x^2}\right)\ dx$

$$= \int (x^2+2+x^{-2})\ dx = \frac{1}{3}x^3 + 2x - x^{-1} + C$$

$$= \frac{1}{3}x^3 + 2x - \frac{1}{x} + C$$

(29) $\int \left(x+\frac{1}{x^2}\right)^2\ dx = \int \left(x^2+\frac{2}{x}+\frac{1}{x^4}\right)\ dx$

$$= \int \left(x^2+\frac{2}{x}+x^{-4}\right)\ dx = \frac{1}{3}x^3 + 2\log x - \frac{1}{3}x^{-3} + C$$

$$= \frac{1}{3}x^3 + 2\log x - \frac{1}{3x^3} + C$$

(30) $\int (e^x+e^{-x})(e^x-e^{-x})\ dx = \int (e^{2x}-e^{-2x})\ dx$

$$= \frac{1}{2}e^{2x} + \frac{1}{2}e^{-2x} + C$$

(31) $\int \frac{e^x-e^{-x}}{e^x+e^{-x}}\ dx = \log(e^x+e^{-x}) + C$

(32) $\int (e^x + e^{-x})^2\,dx = \int (e^{2x} + 2 + e^{-2x})\,dx$

$= \frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + C$

$\left(※\ e^x \times e^{-x} = e^x \times \frac{1}{e^x} = 1 \right)$

(33) $\int xe^{2x}\,dx = \int x\left(\frac{1}{2}e^{2x}\right)'\,dx$

$= \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}\,dx$

$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$

(34) $\int xe^{x^2}\,dx$

$u = x^2 \cdots$① とおくと

$\frac{du}{dx} = 2x \qquad \therefore dx = \frac{du}{2x} \cdots$②

①，②より

(与式) $= \int \cancel{x} e^u \times \frac{du}{2\cancel{x}} = \frac{1}{2}\int e^u\,du$

$= \frac{1}{2}e^u + C$

$= \frac{1}{2}e^{x^2} + C$

(35) $\int x^2 e^x\,dx = \int x^2 (e^x)'\,dx$

$= x^2 e^x - 2\int xe^x\,dx$

$= x^2 e^x - 2\int x(e^x)'\,dx$

$= x^2 e^x - 2\left(xe^x - \int e^x\,dx\right)$

$= x^2 e^x - 2(xe^x - e^x) + C$

$= (x^2 - 2x + 2)e^x + C$

(36) $\int e^{\sin x}\cos x\,dx$

$u = \sin x \cdots$① とおくと

$\frac{du}{dx} = \cos x \qquad \therefore dx = \frac{du}{\cos x} \cdots$②

①，②より

(与式) $= \int e^u \cancel{\cos x} \times \frac{du}{\cancel{\cos x}} = \int e^u\,du$

$= e^u + C$

$= e^{\sin x} + C$

(37) $\int \log x\,dx = \int (x)'\log x\,dx$

$= x\log x - \int \left(x \times \frac{1}{x}\right)\,dx$

$= x\log x - \int dx$

$= x\log x - x + C$

(38) $\int x\log x\,dx = \int \left(\frac{1}{2}x^2\right)'\log x\,dx$

$= \frac{1}{2}x^2\log x - \frac{1}{2}\int \left(x^2 \times \frac{1}{x}\right)\,dx$

$= \frac{1}{2}x^2\log x - \frac{1}{2}\int x\,dx$

$= \frac{1}{2}x^2\log x - \frac{1}{4}x^2 + C$

(39) $\int \frac{\log x}{x}\,dx$

$u = \log x \cdots$① とおくと

$\frac{du}{dx} = \frac{1}{x} \qquad \therefore dx = x\,du \cdots$②

①，②より

(与式) $= \int \frac{u}{\cancel{x}} \times \cancel{x}\,du = \int u\,du$

$= \frac{1}{2}u^2 + C = \frac{1}{2}(\log x)^2 + C$