演習　不定積分_第 11 回

学年[　2　]年　学科[MI・AC・BC]　番号[　　　　]　氏名　[　　　　　　　　　　　　　]

問 9.8　次の不定積分を求めよ。

(1) $\displaystyle\int x\sqrt{x^2+1}\,dx$

$u = x^2+1$ …①　とおくと　$\dfrac{du}{dx} = 2x$　　$\therefore\ dx = \dfrac{du}{2x}$ …②

①，②より　(与式) $\displaystyle = \int \cancel{x}\sqrt{u} \times \frac{du}{2\cancel{x}} = \frac{1}{2}\int u^{\frac{1}{2}}du = \frac{1}{3}u^{\frac{3}{2}} + C = \frac{1}{3}\sqrt{(x^2+1)^3} + C$

(2) $\displaystyle\int \frac{\log x}{x}\,dx$

$u = \log x$ …①　とおくと　$\dfrac{du}{dx} = \dfrac{1}{x}$　　$\therefore\ dx = x\,du$ …②

①，②より　(与式) $\displaystyle = \int \frac{u}{\cancel{x}} \times \cancel{x}\ du = \int u\ du = \frac{1}{2}u^2 + C = \frac{1}{2}(\log x)^2 + C$

(3) $\displaystyle\int \frac{e^{\tan x}}{\cos^2 x}\,dx$

$u = \tan x$ …①　とおくと　$\dfrac{du}{dx} = \dfrac{1}{\cos^2 x}$　　$\therefore\ dx = \cos^2 x\ du$ …②

①，②より　(与式) $\displaystyle = \int \frac{e^u}{\cancel{\cos^2 x}} \times \cancel{\cos^2 x}\ du = \int e^u\ du = e^u + C = e^{\tan x} + C$

(4) $\displaystyle\int \sin x \cos^5 x\,dx$　[※ $\cos^5 x = \{\cos x\}^5$　のことです]

$u = \cos x$ …①　とおくと　$\dfrac{du}{dx} = -\sin x$　　$\therefore\ dx = \dfrac{-du}{\sin x}$ …②

①，②より　(与式) $\displaystyle = \int \cancel{\sin x} \times u^5 \times \frac{-du}{\cancel{\sin x}} = -\int u^5\ du = -\frac{1}{6}u^6 + C = -\frac{1}{6}\cos^6 x + C$

(5) $\displaystyle\int e^x(1+e^x)^4\,dx$

$u = 1+e^x$ …①　とおくと　$\dfrac{du}{dx} = e^x$　　$\therefore\ dx = \dfrac{du}{e^x}$ …②

①，②より　(与式) $\displaystyle = \int \cancel{e^x} \times u^4 \times \frac{du}{\cancel{e^x}} = \int u^4\ du = \frac{1}{5}u^5 + C = \frac{1}{5}(1+e^x)^5 + C$