

問9.6 不定積分  $\int x^2 \cos 2x \, dx$  を求めよ。

$$\begin{aligned}
 \int x^2 \cos 2x \, dx &= \int x^2 \left( \frac{1}{2} \sin 2x \right)' dx \\
 &= \frac{1}{2} x^2 \sin 2x - \int x \sin 2x \, dx \\
 &= \frac{1}{2} x^2 \sin 2x - \int x \left( -\frac{1}{2} \cos 2x \right)' dx \\
 &= \frac{1}{2} x^2 \sin 2x - \left( -\frac{1}{2} x \cos 2x + \frac{1}{2} \int \cos 2x \, dx \right) \\
 &= \frac{1}{2} x^2 \sin 2x - \left( -\frac{1}{2} x \cos 2x + \frac{1}{4} \sin 2x \right) + C \\
 &= \frac{1}{2} x^2 \sin 2x + \frac{1}{2} x \cos 2x - \frac{1}{4} \sin 2x + C
 \end{aligned}$$

問9.7 不定積分  $\int e^{3x} \cos 2x \, dx$  を求めよ。

$$\begin{aligned}
 \int e^{3x} \cos 2x \, dx &= \int \left( \frac{1}{3} e^{3x} \right)' \cos 2x \, dx \\
 &= \frac{1}{3} e^{3x} \cos 2x + \frac{2}{3} \int e^{3x} \sin 2x \, dx \\
 &= \frac{1}{3} e^{3x} \cos 2x + \frac{2}{3} \int \left( \frac{1}{3} e^{3x} \right)' \sin 2x \, dx \\
 &= \frac{1}{3} e^{3x} \cos 2x + \frac{2}{3} \left( \frac{1}{3} e^{3x} \sin 2x - \frac{2}{3} \int e^{3x} \cos 2x \, dx \right) \\
 &= \frac{1}{3} e^{3x} \cos 2x + \frac{2}{9} e^{3x} \sin 2x - \frac{4}{9} \int e^{3x} \cos 2x \, dx
 \end{aligned}$$

よって、次式が成り立つ。

$$\begin{aligned}
 \frac{13}{9} \int e^{3x} \cos 2x \, dx &= \frac{1}{3} e^{3x} \cos 2x + \frac{2}{9} e^{3x} \sin 2x + c \\
 \therefore \int e^{3x} \cos 2x \, dx &= \frac{3}{13} e^{3x} \cos 2x + \frac{2}{13} e^{3x} \sin 2x + C \quad \left( \text{但し } C = \frac{9}{13} c \right)
 \end{aligned}$$