演習　微分_第 05 回

学年[2]年　学科[MI・AC・BC]　番号[　　]　氏名　[　　]

問 4.6　次の極限値を求めよ。

(1) $\lim_{\theta\to 0}\dfrac{\sin 3\theta}{2\theta} = \lim_{\theta\to 0}\dfrac{\sin 3\theta}{\boxed{3\theta}} \times \dfrac{\boxed{3\theta}}{2\theta} = 1\times\dfrac{3}{2} = \dfrac{3}{2}$

(2) $\lim_{\theta\to 0}\dfrac{\tan 5\theta}{\sin 3\theta} = \lim_{\theta\to 0}\dfrac{\tan 5\theta}{\boxed{5\theta}} \times \dfrac{\boxed{3\theta}}{\sin 3\theta} \times \dfrac{\boxed{5\theta}}{\boxed{3\theta}} = 1\times 1\times\dfrac{5}{3} = \dfrac{5}{3}$

(3) $\lim_{\theta\to 0}\dfrac{1-\cos\theta}{\theta^2}$　[Hint：(3)は分母分子に $(1+\cos\theta)$ を掛ける]

$$= \lim_{\theta\to 0}\frac{(1-\cos\theta)(1+\cos\theta)}{\theta^2(1+\cos\theta)} = \lim_{\theta\to 0}\frac{1-\cos^2\theta}{\theta^2(1+\cos\theta)}$$

$$= \lim_{\theta\to 0}\frac{\sin^2\theta}{\theta^2(1+\cos\theta)} = \lim_{\theta\to 0}\left(\frac{\sin\theta}{\theta}\right)^2 \times \frac{1}{1+\cos\theta} = 1^2\times\frac{1}{1+\cos 0} = \frac{1}{2}$$

問 4.7　次の関数を微分せよ。

(1) $y = x\sin x$　　$y' = 1\times\sin x + x\times\cos x = \sin x + x\cos x$

(2) $y = \dfrac{\sin x}{1+\cos x}$　　$y' = \dfrac{\cos x\times(1+\cos x) - \sin x\times(-\sin x)}{(1+\cos x)^2}$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} = \frac{(\cos^2 x + \sin^2 x) + \cos x}{(1+\cos x)^2}$$

$$= \frac{1+\cos x}{(1+\cos x)^2} = \frac{1}{1+\cos x}$$

(3) $y = \tan(2x+1) = \tan u$　（$u = 2x+1$）

$$y' = \frac{1}{\cos^2 u}\times u' = \frac{2}{\cos^2(2x+1)}$$

(4) $y = e^{\sin x} = e^u$　（$u = \sin x$）

$$y' = e^u\times u' = e^{\sin x}\cos x$$

(5) $y = \log|\cos x| = \log|u|$　（$u = \cos x$）

$$y' = \frac{u'}{u} = \frac{-\sin x}{\cos x} = -\tan x$$

(6) $y = (1+\tan x)^3 = u^3$　（$u = 1+\tan x$）

$$y' = 3u^2u' = 3(1+\tan x)^2\times\frac{1}{\cos^2 x} = \frac{3(1+\tan x)^2}{\cos^2 x}$$