演習　微分の応用_第 12回

学年［ 2 ］年　学科［ MI・AC・BC ］　番号［　　　　］　氏名　［　　　　　　　　　　　　　］

次の関数を微分せよ。

(01) $y = x^5$

$y' = 5x^4$

(02) $y = 3x^2 - 5x + 4$

$y' = 6x - 5$

(03) $y = \frac{2}{x} = 2x^{-1}$

$y' = -2x^{-2} = -\frac{2}{x^2}$

(04) $y = \frac{1}{3x^6} = \frac{1}{3}x^{-6}$

$y' = -2x^{-7} = -\frac{2}{x^7}$

(05) $y = \sqrt[3]{x^4} = x^{\frac{4}{3}}$

$y' = \frac{4}{3}x^{\frac{1}{3}} = \frac{4}{3}\sqrt[3]{x}$

(06) $y = \frac{1}{\sqrt[6]{x}} = x^{-\frac{1}{6}}$

$y' = -\frac{1}{6}x^{-\frac{7}{6}} = -\frac{1}{6\sqrt[6]{x^7}}$

(07) $y = \frac{1}{3}x^3 - \frac{1}{x} + 5\sqrt[5]{x} = \frac{1}{3}x^3 - x^{-1} + 5x^{\frac{1}{5}}$

$y' = x^2 + x^{-2} + x^{-\frac{4}{5}} = x^2 + \frac{1}{x^2} + \frac{1}{\sqrt[5]{x^4}}$

(08) $y = (x^2 - 3x + 5)^3$

$y' = 3(x^2 - 3x + 5)^2 \times (2x - 3)$

$= 3(2x - 3)(x^2 - 3x + 5)^2$

(09) $y = \frac{1}{(4x-7)^6} = (4x-7)^{-6}$

$y' = -6(4x-7)^{-7} \times 4 = -\frac{24}{(4x-7)^7}$

(10) $y = \sqrt[3]{3x+5} = (3x+5)^{\frac{1}{3}}$

$y' = \frac{1}{3}(3x+5)^{-\frac{2}{3}} \times 3 = \frac{1}{\sqrt[3]{(3x+5)^2}}$

(11) $y = \frac{2}{x+1}$

$y' = \frac{\cancel{0 \times (x+1)} - 2 \times 1}{(x+1)^2} = -\frac{2}{(x+1)^2}$

(12) $y = \frac{4x+3}{x-2}$

$y' = \frac{4 \times (x-2) - (4x+3) \times 1}{(x-2)^2}$

$= \frac{(4x-8) - (4x+3)}{(x-2)^2} = -\frac{11}{(x-2)^2}$

(13) $y = x^3(x-1)^4$

$y' = 3x^2 \times (x-1)^4 + x^3 \times \{4(x-1)^3 \times 1\}$

$= 3x^2(x-1)^4 + 4x^3(x-1)^3$

$= x^2(x-1)^3\{3(x-1) + 4x\}$

$= x^2(x-1)^3(7x-3)$

(14) $y = \log(3x+5)$

$y' = \frac{3}{3x+5}$

(15) $y = \log(x^2 + x - 1)$

$y' = \frac{2x+1}{x^2 + x - 1}$

(16) $y=\log|4x-3|$

$$y'=\frac{4}{4x-3}$$

(17) $y=\log_3|7x+4|$

$$y'=\frac{7}{(7x+4)\log 3}$$

(18) $y=x^2\log(2x+1)$

$$y'=2x\times\log(2x+1)+x^2\times\frac{2}{2x+1}$$

$$=2x\log(2x+1)+\frac{2x^2}{2x+1}$$

(19) $y=(2x+1)^2\log x$

$$y'=\{2(2x+1)\times 2\}\times\log x+(2x+1)^2\times\frac{1}{x}$$

$$=4(2x+1)\log x+\frac{(2x+1)^2}{x}$$

(20) $y=\dfrac{\log(3x-2)}{x^2}$

$$y'=\frac{\frac{3}{3x-2}\times \cancel{x^2}-\log(3x-2)\times 2\cancel{x}}{\cancel{x^4}\,x^3}$$

$$=\frac{3x-2(3x-2)\log(3x-2)}{x^3(3x-2)}$$

(21) $y=(1+\log x)^3$

$$y'=3(1+\log x)^2\times\frac{1}{x}=\frac{3(1+\log x)^2}{x}$$

(22) $y=3^{2x+1}$

$$y'=2\cdot 3^{2x+1}\cdot\log 3$$

(23) $y=e^{x^2-3x-2}$

$$y'=(2x-3)e^{x^2-3x-2}$$

(24) $y=(x-1)e^{3x}$

$$y'=1\times e^{3x}+(x-1)\times 3e^{3x}=(3x-2)e^{3x}$$

(25) $y=(e^x+e^{-x})^2$

$$y'=2(e^x+e^{-x})(e^x-e^{-x})=2(e^{2x}-e^{-2x})$$

(26) $y=\dfrac{\log x}{e^x}$

$$y'=\frac{\frac{1}{x}\times e^x-\log x\times e^x}{(e^x)^2}$$

$$=\frac{\frac{1}{x}-\log x}{e^x}=\frac{1-x\log x}{xe^x}$$

(27) $y=x\sin x$

$$y'=1\times\sin x+x\times\cos x=\sin x+x\cos x$$

(28) $y=\dfrac{\sin x}{1+\cos x}$

$$y'=\frac{\cos x\times(1+\cos x)-\sin x\times(-\sin x)}{(1+\cos x)^2}$$

$$=\frac{\cos x+\cos^2 x+\sin^2 x}{(1+\cos x)^2}$$

$$=\frac{(\cos^2 x+\sin^2 x)+\cos x}{(1+\cos x)^2}$$

$$=\frac{1+\cos x}{(1+\cos x)^2}=\frac{1}{1+\cos x}$$

(29) $y=\tan(2x+1)$

$$y'=\frac{1}{\cos^2 u}\times u'=\frac{2}{\cos^2(2x+1)}$$

(30) $y=e^{\sin x}$

$$y'=e^u\times u'=e^{\sin x}\cos x$$

(31) $y=\log|\cos x|$

$$y'=\frac{-\sin x}{\cos x}=-\tan x$$

(32) $y=(1+\tan x)^3$

$$y'=3(1+\tan x)^2\times\frac{1}{\cos^2 x}=\frac{3(1+\tan x)^2}{\cos^2 x}$$

(33) $y=\mathrm{Sin}^{-1}3x$

$$y'=\frac{3}{\sqrt{1-(3x)^2}}=\frac{3}{\sqrt{1-9x^2}}$$

(34) $y=\mathrm{Cos}^{-1}\frac{x}{3}$

$$y'=\frac{-\frac{1}{3}}{\sqrt{1-\left(\frac{x}{3}\right)^2}}=\frac{-1}{\sqrt{9-x^2}}$$

(35) $y=\mathrm{Tan}^{-1}\frac{x}{3}$

$$y'=\frac{\frac{1}{3}}{\left(\frac{x}{3}\right)^2+1}=\frac{3}{x^2+9}$$

(36) $y=(2x+3)^4$

$$y'=4(2x+3)^3\times 2=8(2x+3)^3$$

(37) $y=(x^2-x+1)^3$

$$y'=3(x^2-x+1)^2(2x-1)$$

(38) $y=\frac{1}{\cos x}$

$$y'=-\frac{-\sin x}{\cos^2 x}=\frac{\sin x}{\cos^2 x}$$

(39) $y=\log(x^2-x+1)$

$$y'=\frac{2x-1}{x^2-x+1}$$

(40) $y=e^{x^2-x+1}$

$$y'=(2x-1)e^{x^2-x+1}$$

(41) $y=\cos(2x+3)$

$$y'=-2\sin(2x+3)$$

(42) $y=\tan(2x+3)$

$$y'=\frac{2}{\cos^2(2x+3)}$$

(43) $y=e^{2x}\cos 3x$

$$y'=2e^{2x}\times\cos 3x+e^{2x}\times(-3\sin 3x)$$
$$=e^{2x}(2\cos 3x-3\sin 3x)$$

(44) $y=\frac{e^{2x}}{\sin 3x}$

$$y'=\frac{2e^{2x}\times\sin 3x-e^{2x}\times 3\cos 3x}{\sin^2 3x}$$
$$=\frac{e^{2x}(2\sin 3x-3\cos 3x)}{\sin^2 3x}$$

(45) $y=\{\log(2x+3)\}^6=u^6$

$$y'=6u^5\times u'=6\{\log(2x+3)\}^5\times\frac{2}{2x+3}$$
$$=\frac{12\{\log(2x+3)\}^5}{2x+3}$$

(46) $y=\cos^5 2x=u^5$

$$y'=5u^4\times u'=5\cos^4 2x\times(-2\sin 2x)$$
$$=-10\cos^4 2x\sin 2x$$

[Hint] 次の公式を使用する

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

(47) $y = \sqrt{1-x^2} = \sqrt{u}$

$$y' = \frac{u'}{2\sqrt{u}} = \frac{-2x}{2\sqrt{1-x^2}} = \frac{-x}{\sqrt{1-x^2}}$$

(48) $y = \log(x + \sqrt{x^2+1})$

$$y' = \frac{1}{x+\sqrt{x^2+1}} \times \left(1 + \frac{2x}{2\sqrt{x^2+1}}\right)$$

$$= \frac{1}{x+\sqrt{x^2+1}} \times \frac{x+\sqrt{x^2+1}}{\sqrt{x^2+1}}$$

$$= \frac{1}{\sqrt{x^2+1}}$$

[Hint] 対数微分法を使用する

(49) $y = x^x$ （ $x > 0$ ）

対数をとる

$$\log y = \log x^x = x \log x$$

微分する

$$\frac{y'}{y} = 1 \times \log x + x \times \frac{1}{x}$$

$$= \log x + 1$$

よって，求める導関数は

$$y' = (1+\log x)y = (1+\log x)x^x$$

(50) $y = \dfrac{(2x+1)^3}{(4x-1)^2}$

［直接計算］

$$y' = \frac{\{3(2x+1)^2 \times 2\} \times (4x-1)^2 - (2x+1)^3 \times \{2(4x-1) \times 4\}}{(4x-1)^4}$$

$$= \frac{6(2x+1)^2(4x-1)^{\cancel{2}} - 8(2x+1)^3\cancel{(4x-1)}}{(4x-1)^{\cancel{4}3}}$$

$$= \frac{6(2x+1)^2(4x-1) - 8(2x+1)^3}{(4x-1)^3}$$

$$= \frac{2(2x+1)^2\{3(4x-1) - 4(2x+1)\}}{(4x-1)^3}$$

$$= \frac{2(2x+1)^2(12x-3-8x-4)}{(4x-1)^3}$$

$$= \frac{2(2x+1)^2(4x-7)}{(4x-1)^3}$$

［対数微分法］

対数をとる

$$\log y = \log \frac{(2x+1)^3}{(4x-1)^2}$$

$$= \log(2x+1)^3 - \log(4x-1)^2$$

$$= 3\log(2x+1) - 2\log(4x-1)$$

微分する

$$\frac{y'}{y} = 3 \times \frac{2}{2x+1} - 2 \times \frac{4}{4x-1}$$

$$= \frac{6(4x-1) - 8(2x+1)}{(2x+1)(4x-1)}$$

$$= \frac{24x-6-16x-8}{(2x+1)(4x-1)}$$

$$= \frac{8x-14}{(2x+1)(4x-1)} = \frac{2(4x-7)}{(2x+1)(4x-1)}$$

よって，求める導関数は

$$y' = \frac{2(4x-7)}{(2x+1)(4x-1)} \times \frac{(2x+1)^3}{(4x-1)^2}$$

$$= \frac{2(4x-7)(2x+1)^2}{(4x-1)^3}$$