

問 5.5 次の極限を、ロピタルの定理を用いて求めよ。

$$(1) \lim_{x \rightarrow 1} \frac{3x^3 - 4x^2 + 2x - 1}{2x^3 + 3x^2 - x - 4} = \lim_{x \rightarrow 1} \frac{9x^2 - 8x + 2}{6x^2 + 6x - 1} = \frac{3}{11} \quad (\text{収束})$$

$$(2) \lim_{x \rightarrow +\infty} \frac{4x + 3}{x^2 - x + 1} = \lim_{x \rightarrow +\infty} \frac{4}{2x - 1} = \frac{4}{+\infty} = +0 \quad (\text{収束})$$

$$(3) \lim_{x \rightarrow 0} \frac{\tan 3x}{x} = \lim_{x \rightarrow 0} \frac{3}{\cos^2 3x} = \frac{3}{\cos^2 0} = 3 \quad (\text{収束})$$

$$\left[ \text{[Hint] } \textcircled{1} (\tan x)' = \frac{1}{\cos^2 x} \quad \textcircled{2} \cos 0 = 1 \right]$$

$$(4) \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \lim_{x \rightarrow 0} \frac{\sin x}{2x} \left( = \frac{0}{0} : \text{不定形} \right)$$

$$= \lim_{x \rightarrow 0} \frac{\cos x}{2} = \frac{\cos 0}{2} = \frac{1}{2} \quad (\text{収束})$$

$$(5) \lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 2x} = \lim_{x \rightarrow +\infty} \frac{3 \cos 3x}{2 \cos 2x} = \frac{3 \cos 0}{2 \cos 0} = \frac{3}{2} \quad (\text{収束})$$

$$(6) \lim_{x \rightarrow +\infty} \frac{\log x}{x} = \lim_{x \rightarrow +\infty} \frac{1}{x} = \frac{1}{+\infty} = +0 \quad (\text{収束})$$

$$(7) \lim_{x \rightarrow +\infty} \frac{e^x}{x^2} = \lim_{x \rightarrow +\infty} \frac{e^x}{2x} \left( = \frac{\infty}{\infty} : \text{不定形} \right)$$

$$= \lim_{x \rightarrow +\infty} \frac{e^x}{2} = \frac{+\infty}{2} = +\infty \quad (\text{発散})$$