3. 三角関数

$$180^\circ = \pi\,(\text{rad})$$

問　次の問いに答えよ。

(1) $2\,(\text{rad}) = x^\circ \Rightarrow x =$ $\dfrac{2\,(\text{rad})}{\pi\,(\text{rad})} = \dfrac{x^\circ}{180^\circ} \Rightarrow x = 180 \times \dfrac{2}{\pi} = \dfrac{360}{\pi}$

(2) $y\,(\text{rad}) = 12^\circ \Rightarrow y =$ $\dfrac{y\,(\text{rad})}{\pi\,(\text{rad})} = \dfrac{12^\circ}{180^\circ} \Rightarrow y = \dfrac{12}{180} \times \pi = \dfrac{2}{30} \times \pi = \dfrac{\pi}{15}$

$$y = a \underset{\cos}{\sin} bx + c \Rightarrow (\text{振幅}) = a,\ (\text{周期}) = \frac{2\pi}{b}、\ y = \tan bx \Rightarrow (\text{周期}) = \frac{\pi}{b}$$

グラフ

問　振幅、周期等を調べて次のグラフを書け。

(1) $y = \sin\dfrac{x}{2} + 1$　振幅＝ 1 、周期＝ $2\pi \div \dfrac{1}{2} = 4\pi$ 、「水平線」 $y = 1$　グラフ省略

(2) $y = \sin^2 x = \dfrac{1}{2}(1 - \cos 2x) = -\dfrac{1}{2}\cos 2x + \dfrac{1}{2}$

振幅＝ $\left|-\dfrac{1}{2}\right| = \dfrac{1}{2}$、 周期＝ $2\pi \div 2 = \pi$ 、「水平線」 $y = \dfrac{1}{2}$　グラフ省略

(3) $y = \tan 2x$　周期＝ $\pi \div 2 = \dfrac{\pi}{2}$　グラフ省略

三角方程式・不等式

問　次の方程式・不等式を解け。

(1) $\sin x = -\dfrac{1}{2}$ $(0 \leqq x < 2\pi)$

単位円と直線 $y = \dfrac{-1}{2}$ の交点に対応する角なので　$x = 210^\circ, 330^\circ = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

(2) $\cos x \leqq -\dfrac{1}{2}$ $(0 \leqq x < 2\pi)$

単位円の直線 $x = \dfrac{-1}{2}$ のより左側にある部分に対応する角なので　$120^\circ \leqq x \leqq 240^\circ \Rightarrow \dfrac{2\pi}{3} \leqq x \leqq \dfrac{4\pi}{3}$

(3) $\tan\left(x - \dfrac{\pi}{4}\right) = \sqrt{3}$ $(0 \leqq x < \pi)$

$X = x - \dfrac{\pi}{4}$ とおくと、$0 \leqq x < \pi \Rightarrow -\dfrac{\pi}{4} \leqq x - \dfrac{\pi}{4} < \dfrac{3\pi}{4} \Rightarrow -\dfrac{\pi}{4} \leqq X < \dfrac{3\pi}{4}$ だから、

この範囲で傾きが $\sqrt{3}$ の原点を通る直線と x 軸のなす角 X は、$X = x - \dfrac{\pi}{4} = \dfrac{\pi}{3} \Rightarrow x = \dfrac{\pi}{3} + \dfrac{\pi}{4} = \dfrac{7\pi}{12}$

基本公式と加法定理

$\sin^2 x = 1-\cos^2 x \left(\cos^2 x = 1-\sin^2 x\right)$　$1+\tan^2 x = \dfrac{1}{\cos^2 x}\left(\cos^2 x+\sin^2 x = 1,\ \tan x = \dfrac{\sin x}{\cos x}\right)$

$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta\pm\cos\alpha\sin\beta$　$\cos^2 x = \dfrac{1}{1+\tan^2 x}$

$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta\mp\sin\alpha\sin\beta$　$\tan(\alpha\pm\beta) = \dfrac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}$

$\sin 2x = 2\sin x\cos x$　$\cos 2x = \cos^2 x-\sin^2 x$　$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

$\sin^2 x = \dfrac{1-\cos 2x}{2}$　$\cos^2 x = \dfrac{1+\cos 2x}{2}$　$a\sin x+b\cos x = \sqrt{a^2+b^2}\sin(x+\theta)\ \left(\tan\theta = \dfrac{b}{a}\right)$

$\sin A+\sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A+B}{2}$　$\sin\alpha\cos\beta = \dfrac{1}{2}\{\sin(\alpha+\beta)+\sin(\alpha-\beta)\}$

$\sin A-\sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A+B}{2}$　$\cos\alpha\sin\beta = \dfrac{1}{2}\{\sin(\alpha+\beta)-\sin(\alpha-\beta)\}$

$\cos A+\cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A+B}{2}$　$\cos\alpha\cos\beta = \dfrac{1}{2}\{\cos(\alpha+\beta)+\cos(\alpha-\beta)\}$

$\cos A-\cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A+B}{2}$　$\sin\alpha\sin\beta = \dfrac{-1}{2}\{\cos(\alpha+\beta)-\cos(\alpha-\beta)\}$

問 α は第 2 象限の角で　$\cos\alpha = -\dfrac{1}{3}$、　β は第 3 象限の角で　$\tan\beta = 2\sqrt{2}$　のとき、次を求めよ。

(1) 残りの三角関数の値　α は第 2 象限の角なので $\sin\alpha > 0$、β は第 3 象限の角なので $\cos\beta < 0$

$\cos\alpha = \dfrac{-1}{3}$　　$\cos\beta = -\sqrt{\dfrac{1}{1+\tan^2\beta}} = -\sqrt{\dfrac{1}{1+8}} = \dfrac{-1}{3}$

$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\dfrac{1}{9}} = \dfrac{2\sqrt{2}}{3}$　　$\sin\beta = \tan\beta\cos\beta = 2\sqrt{2}\times\dfrac{-1}{3} = \dfrac{-2\sqrt{2}}{3}$

$\tan\alpha = \sin\alpha\times\dfrac{1}{\cos\alpha} = \dfrac{2\sqrt{2}}{3}\times\dfrac{3}{-1} = -2\sqrt{2}$　　$\tan\beta = 2\sqrt{2}$

(2) $\sin(\alpha+\beta) = \sin\alpha\cos\beta+\cos\alpha\sin\alpha = \dfrac{2\sqrt{2}}{3}\cdot\dfrac{-1}{3}+\dfrac{-1}{3}\cdot\dfrac{-2\sqrt{2}}{3} = 0$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta-\sin\alpha\sin\beta = \dfrac{-1}{3}\cdot\dfrac{-1}{3}-\dfrac{2\sqrt{2}}{3}\cdot\dfrac{-2\sqrt{2}}{3} = 1$

$\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \dfrac{2\sqrt{2}+\left(-2\sqrt{2}\right)}{1-2\sqrt{2}\left(-2\sqrt{2}\right)} = 0$　$\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\cdot\dfrac{2\sqrt{2}}{3}\cdot\dfrac{-1}{3} = \dfrac{-4\sqrt{2}}{9}$

$\cos 2\alpha = \cos^2\alpha-\sin^2\alpha = \dfrac{1}{9}-\dfrac{8}{9} = \dfrac{-7}{9}$　　$\tan 2\beta = \dfrac{2\tan\beta}{1-\tan^2\beta} = \dfrac{4\sqrt{2}}{1-8} = \dfrac{-4\sqrt{2}}{7}$

$\sin^2\dfrac{\alpha}{2} = \dfrac{1}{2}(1-\cos\alpha) = \dfrac{1}{2}\left(1-\dfrac{-1}{3}\right) = \dfrac{2}{3}$　　$\cos^2\dfrac{\beta}{2} = \dfrac{1}{2}(1+\cos\alpha) = \dfrac{1}{2}\left(1+\dfrac{-1}{3}\right) = \dfrac{1}{3}$

(3) $\sqrt{3}\sin x-\cos x = r\sin(x+\theta) \Rightarrow r = \sqrt{\left(\sqrt{3}\right)^2+(-1)^2} = 2$

係数は $\left(\sqrt{3},-1\right)$（第 4 象限の点）なので、$\theta = \mathrm{Tan}^{-1}\dfrac{-1}{\sqrt{3}} = -\mathrm{Tan}^{-1}\dfrac{1}{\sqrt{3}} = \dfrac{-\pi}{6}$