

問 2.5 次の極限の収束・発散を調べよ。収束する場合は極限值を求めよ。

$$(1) \lim_{x \rightarrow +\infty} (x^2 - x + 1) = \lim_{x \rightarrow +\infty} x^2 \left(1 - \frac{1}{x} + \frac{1}{x^2} \right) = +\infty \times (1 - 0 + 0) = +\infty \quad (\text{発散})$$

$$(2) \lim_{x \rightarrow +\infty} \frac{2x^2 - 3x + 1}{3x^2 + x - 2} = \lim_{x \rightarrow +\infty} \frac{x^2 \left(2 - \frac{3}{x} + \frac{1}{x^2} \right)}{x^2 \left(3 + \frac{1}{x} - \frac{2}{x^2} \right)} = \lim_{x \rightarrow +\infty} \frac{\left(2 - \frac{3}{x} + \frac{1}{x^2} \right)}{\left(3 + \frac{1}{x} - \frac{2}{x^2} \right)} = \frac{2 - 0 + 0}{3 + 0 - 0} = \frac{2}{3} \quad (\text{収束})$$

$$(3) \lim_{x \rightarrow +\infty} \frac{x^3 + 1}{x^2 + x + 1} = \lim_{x \rightarrow +\infty} \frac{x^3 \left(1 + \frac{1}{x^3} \right)}{x^2 \left(1 + \frac{1}{x} + \frac{1}{x^2} \right)} = \lim_{x \rightarrow +\infty} x \times \frac{\left(1 + \frac{1}{x^3} \right)}{\left(1 + \frac{1}{x} + \frac{1}{x^2} \right)} = +\infty \times \frac{1 + 0}{1 + 0 + 0} = +\infty \quad (\text{発散})$$

$$(4) \lim_{x \rightarrow +\infty} \frac{2x + 3}{x^2 - 1} = \lim_{x \rightarrow +\infty} \frac{x \left(2 + \frac{3}{x} \right)}{x^2 \left(1 - \frac{1}{x^2} \right)} = \lim_{x \rightarrow +\infty} \frac{1}{x} \times \frac{\left(2 + \frac{3}{x} \right)}{\left(1 - \frac{1}{x^2} \right)} = +0 \times \frac{2 + 0}{1 - 0} = +0 \quad (\text{収束})$$

問 2.6 次の極限の収束・発散を調べよ。収束する場合は極限值を求めよ。

$$(1) \lim_{x \rightarrow +\infty} (\sqrt{2x+1} - \sqrt{2x-1}) = \lim_{x \rightarrow +\infty} \frac{(\sqrt{2x+1} - \sqrt{2x-1})(\sqrt{2x+1} + \sqrt{2x-1})}{(\sqrt{2x+1} + \sqrt{2x-1})}$$

$$= \lim_{x \rightarrow +\infty} \frac{(2x+1) - (2x-1)}{\sqrt{2x+1} + \sqrt{2x-1}}$$

$$= \lim_{x \rightarrow +\infty} \frac{2}{\sqrt{2x+1} + \sqrt{2x-1}} = \frac{2}{\infty + \infty} = +0 \quad (\text{収束})$$

$$(2) \lim_{x \rightarrow +\infty} (\sqrt{x^2 + x + 1} - \sqrt{x^2 + 1}) = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + x + 1} - \sqrt{x^2 + 1})(\sqrt{x^2 + x + 1} + \sqrt{x^2 + 1})}{(\sqrt{x^2 + x + 1} + \sqrt{x^2 + 1})}$$

$$= \lim_{x \rightarrow +\infty} \frac{(x^2 + x + 1) - (x^2 + 1)}{\sqrt{x^2 + x + 1} + \sqrt{x^2 + 1}} = \lim_{x \rightarrow +\infty} \frac{x}{\sqrt{x^2 + x + 1} + \sqrt{x^2 + 1}}$$

$$= \lim_{x \rightarrow +\infty} \frac{x}{x \left(\sqrt{1 + \frac{1}{x} + \frac{1}{x^2}} + \sqrt{1 + \frac{1}{x^2}} \right)} = \lim_{x \rightarrow +\infty} \frac{1}{\left(\sqrt{1 + \frac{1}{x} + \frac{1}{x^2}} + \sqrt{1 + \frac{1}{x^2}} \right)}$$

$$= \lim_{x \rightarrow +\infty} \frac{1}{\sqrt{1+0+0} + \sqrt{1+0}} = \frac{1}{2} \quad (\text{収束})$$