

問 1.7 次の和を求めよ。

$$(1) \sum_{k=1}^n (2k-1) = 2 \sum_{k=1}^n k - \sum_{k=1}^n 1 = 2 \times \frac{1}{2} n(n+1) - n = n^2 + n - n = n^2$$

$$(2) \sum_{k=1}^n k(k+1) = \sum_{k=1}^n (k^2 + k) = \sum_{k=1}^n k^2 + \sum_{k=1}^n k = \frac{1}{6} n(n+1)(2n+1) + \frac{1}{2} n(n+1)$$

$$= \frac{1}{6} n(n+1)\{(2n+1)+3\} = \frac{1}{6} n(n+1)(2n+4) = \frac{1}{3} n(n+1)(n+2)$$

問 1.8 次の等差数列の和を求めよ。

(1) $-3+1+5+\dots+(\text{第}n\text{項})$

一般項 $a_n = -3 + (n-1) \times 4 = 4n - 7$

①初項 $a_1 = -3$ ②末項 $a_n = 4n - 7$ ③項数 n

よって、求める和は $S_n = \frac{n}{2} \{(-3) + (4n - 7)\} = \frac{1}{2} n(4n - 10) = n(2n - 5)$

(2) $1+3+5+\dots+(\text{第}13\text{項})$

一般項 $a_n = 1 + (n-1) \times 2 = 2n - 1$

①初項 $a_1 = 1$ ②末項 $a_{13} = 2 \times 13 - 1 = 25$ ③項数 $n = 13$

よって、求める和は $S_{13} = \frac{13}{2} (1 + 25) = \frac{13}{2} \times 26 = 169$

(3) $56+\dots+1-4-9$

一般項 $a_n = 56 + (n-1) \times (-5) = 61 - 5n$

①初項 $a = 56$ ②末項 $b = -9$

③項数 $61 - 5n = -9$ より $-5n = -70$ $\therefore n = 14$

よって、求める和は $S_{14} = \frac{14}{2} (56 - 9) = 7 \times 47 = 329$