

問 2.14 次の関数を微分せよ。

$$(1) \quad y = (5x-3)^4 \quad u = 5x-3 \quad \text{とおくと} \quad y = u^4$$

$$y' = 4u^3 \times u' = 4(5x-3)^3 \times 5 = 20(5x-3)^3$$

$$(2) \quad y = (3x+2)^8 \quad u = 3x+2 \quad \text{とおくと} \quad y = u^8$$

$$y' = 8u^7 \times u' = 8(3x+2)^7 \times 3 = 24(3x+2)^7$$

$$(3) \quad y = (x^2 - 3x + 5)^3 \quad u = x^2 - 3x + 5 \quad \text{とおくと} \quad y = u^3$$

$$y' = 3u^2 \times u' = 3(x^2 - 3x + 5)^2 \times (2x - 3)$$

$$= 3(2x - 3)(x^2 - 3x + 5)^2$$

$$(4) \quad y = \frac{1}{(4x-7)^6} \quad u = 4x-7 \quad \text{とおくと} \quad y = \frac{1}{u^6} = u^{-6}$$

$$y' = -6u^{-7} \times u' = -6(4x-7)^{-7} \times 4 = -\frac{24}{(4x-7)^7}$$

問 2.15 次の関数を微分せよ。

$$(1) \quad y = \sqrt[3]{x^4} = x^{\frac{4}{3}} \quad y' = \frac{4}{3}x^{\frac{1}{3}} = \frac{4}{3}\sqrt[3]{x}$$

$$(2) \quad y = \sqrt[4]{x^3} = x^{\frac{3}{4}} \quad y' = \frac{3}{4}x^{-\frac{1}{4}} = \frac{3}{4\sqrt[4]{x}}$$

$$(3) \quad y = x\sqrt{x} = x^1 \times x^{\frac{1}{2}} = x^{1+\frac{1}{2}} = x^{\frac{3}{2}} \quad y' = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$$

$$(4) \quad y = \frac{1}{3}x^3 - \frac{1}{x} + 5\sqrt[5]{x} = \frac{1}{3}x^3 - x^{-1} + 5x^{\frac{1}{5}} \quad y' = x^2 + x^{-2} + x^{-\frac{4}{5}} = x^2 + \frac{1}{x^2} + \frac{1}{\sqrt[5]{x^4}}$$

$$(5) \quad y = (\sqrt[3]{x} + 3)(\sqrt[3]{x^2} - 1) = (x^{\frac{1}{3}} + 3)(x^{\frac{2}{3}} - 1) = x + 3x^{\frac{2}{3}} - x^{\frac{1}{3}} - 3$$

$$y' = 1 + 2x^{-\frac{1}{3}} - \frac{1}{3}x^{-\frac{2}{3}} = 1 + \frac{2}{\sqrt[3]{x}} - \frac{1}{3\sqrt[3]{x^2}}$$

問 2.16 合成関数  $y = \sqrt[3]{3x+5}$  を微分せよ。

$$u = 3x+5 \quad \text{とおくと} \quad y = \sqrt[3]{u} = u^{\frac{1}{3}}$$

$$y' = \frac{1}{3}u^{-\frac{2}{3}} \times u' = \frac{1}{3\sqrt[3]{u^2}} \times 3 = \frac{1}{\sqrt[3]{(3x+5)^2}}$$